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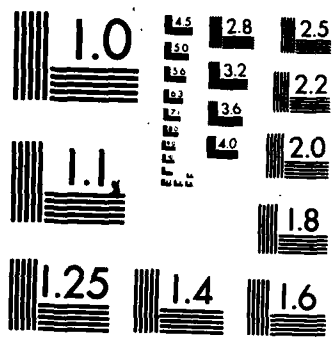
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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This is a progress report of the Applied Mathematics Group in the Mathematics Department, Stanford University. This group began functioning officially on 1 September 1979 and is supported by the Office of Naval Research, the National Science Foundation, the Army Research Office, the Air Force Office of Scientific Research, and Stanford University. The personnel comprising this group during all or part of the reporting periods are Joseph B. Keller, Professor of Mathematics, Stanford University; Russel E. Caflisch, Assistant Professor, Stanford University; John C. Neu, Assistant Professor, Stanford University; Philip F. Rhodes-Robinson, Professor of Mathematics, Victoria University of Wellington; Victor Twersky, Professor of Mathematics, University of Illinois, Chicago Circle; John G. Watson, Assistant Professor, University of Miami; Gary Lieberman, Assistant Professor, Iowa State University; Bernard A. Lippmann, Professor of Physics, Emeritus, New York University; Si-Xiong Chen, Head, Applied Mathematics Group, (CONTINUED

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ITEM #19, ABSTRACT, CONTINUED: Institute of Mechanics, Beijing, People's Republic of China; Meira Falkovitz, Postdoctoral Fellow, Stanford University; John H. Maddocks, Postdoctoral Fellow, Stanford University; Allan D. Jepson, Postdoctoral Fellow, Stanford University; Kevin C. Nunan, Graduate Student. The various research activities of the members of this group are indicated by the list of publications contained in Section II. Abstracts of these publications are contained in Section III. A description of some of the research activities which have been completed but not yet submitted for publication, or which are still in progress, is contained in Section IV.

AFOSR-TR- 84-0032

PROGRESS REPORT, Grant AFOSR-79-0134

September 1, 1981 - August 31, 1982

Applied Mathematics Group
Department of Mathematics
Stanford University

Principal Investigator: Joseph B. Keller

I. Introduction.

This is a progress report of the Applied Mathematics Group in the Mathematics Department, Stanford University. This group began functioning officially on September 1, 1979, and is supported by:

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8. Bernard A. Lippmann, Professor of Physics, Emeritus, New York University;

9. Si-Xiong Chen, Head, Applied Mathematics Group, Institute of Mechanics, Beijing, People's Republic of China;

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11. John H. Maddocks, Postdoctoral Fellow, Stanford University;

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II. PUBLICATIONS OF THE APPLIED MATHEMATICS GROUP

1	J. B. Keller	Training in applied mathematics Pub: Proc. of the Conf. on Graduate Training in Math., T. L. Sherman, ed., Rocky Mountain Mathematics Consortium, Tempe, Arizona, 1979, pp. 110-113.
2	J.-M. Vanden-Broeck L. W. Schwartz	Numerical computation of steep gravity waves in shallow water Pub: Phys. Fluids, 22, 1868-1871, 1979.
3	H. McMaken J. D. Achenbach L. Adler D. K. Lewis	Diffraction of ultrasonic waves by penny-shaped cracks in metals: Theory and experiment Pub: J. Acoust. Soc. Am., 66, 1848-1856, 1979.
4	J.-M. Vanden-Broeck J. B. Keller	A new family of capillary waves Pub: J. Fluid Mech., 98, 161-169, 1980.
5	J.-M. Vanden-Broeck	Nonlinear stern waves Pub: J. Fluid Mech., 93, 603-611, 1980.
6	J. B. Keller	Plate failure under pressure Pub: SIAM Rev., 22, 227-228, 1980.
7	J.-M. Vanden-Broeck	Advances in the numerical computation of capillary-gravity waves Pub: Nonlinear Partial Differential Equations in Engineering and Applied Science, R. L. Sternberg, A. J. Kalinowski and J. S. Papadakis, eds., Marcel Dekker, New York, 1980, pp. 299-310.
8*	S. Childress J. B. Keller	Lichen growth Pub: J. Theor. Biol., 82, 157-163, 1980.

*Not supported by AFOSR.

9	J. B. Keller	Liesegang rings and a theory of fast reaction and slow diffusion Pub: Dynamics and Modelling of Reactive Systems, W. Stewart, ed., Academic, New York, 1980, pp. 211-224.
10	J. B. Keller	Darcy's law for flow in porous media and the two-space method Pub: Nonlinear Partial Differential Equations in Engineering and Applied Science, R. L. Sternberg, A. J. Kalinowski and J. S. Papadakis, eds., Marcel Dekker, New York, 1980, pp. 429-443.
11	R. E. Caflisch	The Boltzmann equation with a soft potential, Part I: Linear, spatially-homogeneous Pub: Comm. Math. Phys., 74, 71-95, 1980.
12	R. E. Caflisch	The Boltzmann equation with a soft potential, Part II: Nonlinear, spatially-periodic Pub: Comm. Math. Phys., 74, 97-109, 1980.
13	J. C. Neu	The method of near identity transformations and applications Pub: SIAM J. Appl. Math., 38, 189-208, 1980.
14	J. C. Neu	Large populations of chemical oscillators Pub: SIAM J. Appl. Math., 38, 305-316, 1980.
15	J. B. Keller M. Miksis	Bubble oscillations of large amplitude Pub: J. Acoust. Soc. Am., 68, 628-633, 1980.
16	J.-M. Vanden-Broeck J. B. Keller	Bubble or drop distortion in a straining flow in two dimensions Pub: Phys. Fluids, 23, 1491-1495, 1980.
17	J. B. Keller	Some bubble and contact problems Pub: SIAM Rev., 22, 442-458, 1980.
18	J.-M. Vanden-Broeck J. B. Keller	Deformation of a bubble or drop in a uniform flow Pub: J. Fluid Mech., 101, 673-686, 1980.

19	J.-M. Vanden-Broeck	Nonlinear gravity-capillary stern waves Pub: Phys. Fluids, 23, 1949-1953, 1980.
20	R. E. Caflisch	An inverse problem for Toeplitz matrices and the synthesis of discrete transmission lines Pub: J. Linear Algebra and Its Applications, 38, 255-272, 1980.
21	R. E. Caflisch	The fluid dynamic limit of the nonlinear Boltzmann equation Pub: Comm. Pure Appl. Math., 33, 651-666, 1980.
22*	J. B. Keller	Tendril shape and lichen growth Pub: Some Mathematical Questions in Biology, Lectures on Math. in the Life Sciences, Vol. 13, Am. Math. Soc., Providence, 1980, pp. 257-274.
23	R. E. Caflisch C. Peskin G. Majda G. Strumolo	Distortion of the arterial pulse Pub: Math. Bio. Sci., 51, 229-260, 1980.
24	P. S. Hagan H. Simpson D. S. Cohen	Spatial structures in predator-prey communities with hereditary effects and diffusion Pub: Math. Biosci., 44, 167-177, 1979.
25	J.-M. Vanden-Broeck	Numerical calculation of gravity-capillary interfacial waves of finite amplitude Pub: Phys. Fluids, 23, 1723-1726, 1980.
26	J.-M. Vanden-Broeck	Two-dimensional drops in slow viscous flow Pub: Phys. Fluids, 24, 175-176, 1981.
27	P. S. Hagan	The instability of non-monotonic wave solutions of parabolic equations Pub: Stud. Appl. Math., 64, 57-88, 1981.

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*Not supported by AFOSR.

28	J. B. Keller J.-M. Vanden-Broeck	Shape of a sail in a flow Pub: Phys. Fluids, 24, 552-553, 1981.
29	J. B. Keller	Temperley's model of gas condensation Pub: J. Chem. Phys., 74, 4203-4204, 1981.
30	J. B. Keller J. G. Watson	Kelvin wave production Pub: J. Phys. Ocean., 11, 284-285, 1981.
31	J. B. Keller S. I. Rubinow	Recurrent precipitation and Liesegang rings Pub: J. Chem. Phys., 74, 5000-5007, 1981.
32	J. B. Keller M. J. Miksis J.-M. Vanden-Broeck	Axisymmetric bubble or drop in a uniform flow Pub: J. Fluid Mech., 108, 89-100, 1981.
33	J.-M. Vanden-Broeck	Numerical calculation of standing waves in water of arbitrary uniform depth Pub: Phys. Fluids, 24, 812-815, 1981.
34	R. E. Caflisch	Evaluation of a function at infinity from its power series Pub: Phys. Rev. Letters, 46, 1255-1256, 1981.
35	J. B. Keller D. M. Levy D. S. Ahluwalia	Internal and surface wave production in a stratified fluid Pub: Wave Motion, 3, 215-229, 1981.
36	R. E. Caflisch J. B. Keller	Quench front propagation Pub: Nuc. Eng. Design, 65, 97-102, 1981.
37	M. J. Miksis	A bubble in an axially symmetric shear flow Pub: Phys. Fluids, 24, 1229-1231, 1981.
38	J.-M. Vanden-Broeck	Deformation of a liquid surface by an impinging gas jet Pub: SIAM J. Appl. Math., 41, 306-309, 1981.

39	J. B. Keller	Oblique derivative boundary conditions and the image method Pub: SIAM J. Appl. Math., 41, 294-300, 1981.
40	R. Burridge J. B. Keller	Poroelasticity equations derived from microstructure Pub: J. Acoust. Soc. Am., 70, 1140-1146, 1981.
41	J. C. Neu	Stochastically perturbed resonance Pub: SIAM J. Appl. Math., 41, 365-369, 1981.
42	M. J. Miksis	Shape of a drop in an electric field Pub: Phys. Fluids, 24, 1967-1972, 1981.
43	J.-M. Vanden-Broeck	The influence of capillarity on cavitating flow past a flat plate Pub: Quart. J. Mech. Appl. Math., 34, 465-473, 1981.
44	A. Jeffrey J. Mvungi	A note on the effect of submerged obstacles on water waves in a channel Pub: J. Appl. Math. Phys., 32, 756-763, 1981.
45	P. S. Hagan	Target patterns in reaction-diffusion systems Pub: Adv. Appl. Math., 2, 400-416, 1981.
46	P. S. Hagan M. S. Cohen	Diffusion induced morphogenesis in the development of Dictoystelium Pub: J. Theor. Biol., 93, 881-908, 1981.
47	P. S. Hagan D. Z. Ting J. D. Doll	Nuclear magnetic-resonance studies of cation-transport across vesicle bilayer membranes Pub: Biophys. J., 34, 189-214, 1981.
48	A. Jeffrey T. Kawahara	A note on the multiple scale Fourier transform Acc: Nonlinear Anal., in press.

49	J. C. Neu	Nonlinear interfacial waves Acc: Phys. Fluids, in press.
50	J. B. Keller	Optimum inspection policies Acc: Management Sci., in press.
51	P. S. Hagan	Travelling and stacked travelling wave solutions of parabolic equations Acc: SIAM J. Math. Anal., in press.
52	J.-M. Vanden-Broeck	Contact problems involving the flow past an inflated aerofoil Acc: J. Appl. Mech., in press.
53	J.-M. Vanden-Broeck	Nonlinear two-dimensional sail theory Acc: Phys. Fluids, in press.
54	J. C. Neu	Convective flow with subcritical instability Acc: Phys. Fluids, in press.
55	P. S. Hagan	Spiral waves in reaction diffusion equations Acc: SIAM J. Appl. Math., in press.
56	J.-M. Vanden-Broeck J. B. Keller	Parabolic approximations for ship waves and wave resistance Acc: Proceedings of the Third International Conference on Numerical Ship Hydrodynamics, Paris, France, June 16-19, 1981.
57	A. Jeffrey T. Kawahara	Asymptotic Methods in Nonlinear Wave Problems Acc: Pitman Publishing, Ltd., London, in press.
58	M. J. Miksis J.-M. Vanden-Broeck J. B. Keller	Rising bubbles Acc: J. Fluid Mech., in press.
59	J. C. Neu	Resonantly interacting waves Acc: SIAM J. Appl. Math., in press.

60	J. B. Keller M. J. Miksis	Surface tension driven flows Acc: SIAM J. Appl. Math., in press.
61	A. Jepson A. Spence	Folds in solutions of two parameter systems and their calculation: Part I Acc: Stanford Univ. Numer. Anal. Report, in press.
62	J. B. Keller	Time-dependent queues Acc: SIAM Rev.
63	R. E. Caflisch B. Nicolaenko	Shock profile solutions of the Boltzmann equation Sub: Comm. Math. Phys.
64	P. S. Hagan R. E. Caflisch J. B. Keller	Arrow's model of optimal pricing, use and exploration of undertain natural resources Sub: Econometrica
65	R. E. Caflisch	Radiation transport in a hot plasma Sub: SIAM J. Appl. Math.
66	J. B. Keller J.-M. Vanden-Broeck	Jets rising and falling under gravity Sub: J. Fluid Mech.
67	R. E. Caflisch	Fluid dynamics and the Boltzmann equation Acc: Stud. Stat. Mech., to appear.
68	M. S. Falkovitz M. Seul H. L. Frisch H. M. McConnell	Theory of periodic structures in lipid bilayer membranes Sub: Proc. Nat. Acad. Sci.
69	R. E. Caflisch	The fluid-dynamic limit of a model Boltzmann equation in the presence of a shock Pub: Institut National de Recherche en Informatique et en Automatique, INRIA No. 81, June 1981, 1-34.

III. ABSTRACTS OF MANUSCRIPTS SUBMITTED SINCE [illegible] 1, 1981.

62. "Time-dependent queues" by Joseph B. Keller, SIAM Rev., submitted.

A single server queue is considered having exponentially distributed inter-arrival and service times with slowly changing time-dependent rates $\lambda(\varepsilon t)$ and $\mu(\varepsilon t)$. The parameter is the ratio of an average inter-arrival time to the time over which the rates change appreciably, so it is small. Therefore an asymptotic solution, valid for ε small, is constructed for the time-dependent queue length probability distribution. It consists of five typical parts corresponding to five typical time periods. They are the initial period, the period of light traffic when $\lambda/\mu < 1$, the saturation transition period when λ/μ increases through unity, the oversaturation period when λ/μ starts out greater than unity and then decreases below unity, and the transition period at the end of oversaturation, when the queue returns to the light traffic condition. By combining the solutions for these five intervals, the solution for any queue with slowly varying rates can be obtained. Some of these parts were found previously by G. F. Newell, and some of the formal results have been shown to be asymptotic by W. A. Massey.

63. "Shock profile solutions of the Boltzmann equation" by R. E. Caflisch and B. Nicolaenko, Comm. Math. Phys., submitted.

Shock waves in gas dynamics can be described by the Euler Navier-Stokes, or Boltzmann equations. We prove the existence of shock profile solutions of the Boltzmann equation for shocks which are weak. The shock

is written as a truncated expansion in powers of the shock strength, the first two terms of which come exactly from the Taylor tanh(x) profile for the Navier-Stokes solution. The full solution is found by a projection method like the Lyapunov-Schmidt method as a bifurcation from the constant state in which the bifurcation parameter is the difference between the speed of sound c_0 and the shock speed s.

64. "Arrow's model of optimal pricing, use and exploration of uncertain natural resources" by P. S. Hagan, R. E. Caflisch, and J. B. Keller, Econometrics, submitted.

Arrow's model for finding the optimal rates of exploration, consumption and pricing of a randomly distributed natural resource is analyzed. At first the model is modified so that each discovery reveals an arbitrary amount of resource. Then it is analyzed asymptotically when this amount is small, and analyzed approximately when the amount is medium or large. In both cases explicit results for the optimal exploration, consumption, and pricing policies are obtained. Finally a numerical method is devised which is applicable whatever the size of the discoveries. The results using it are found to agree with the analytical results in both cases.

65. "Radiation transport in a hot plasma" by Russel E. Caflisch, SIAM J. Appl. Math., submitted.

In various astronomical events, Compton scattering is the dominant interaction mechanism between the radiation and electron fields in a plasma. As a result the radiation distribution rapidly becomes a Bose-Einstein given by $\lambda^{-2}(\alpha e^{\lambda^{-1}} - 1)^{-1}$ with λ the wavelength and $\alpha \geq 1$, a time dependent constant. However due to the divergence of the Bremstrahlung emission spectrum at long wavelengths, the distribution must be Planckian there. In between is a transition region. Based on these principles an expansion is found which is an asymptotically exact solution of the Fokker-Planck equation. Then a simple equation for the time evolution of the radiation temperature is obtained. It is solved numerically and compares well with previous methods.

66. "Jets rising and falling under gravity" by J. B. Keller and J.-M. Vanden-Broeck, J. Fluid Mech., submitted.

Steady two dimensional jets of inviscid incompressible fluid, rising and falling under gravity, are calculated numerically. The shape of each jet depends upon a single parameter, the Froude number $\lambda = q_c(Qg)^{-1/3}$, which ranges from zero to infinity. Here q_c is the velocity at the crest of the jet, i.e. the highest point of the upper surface, Q is the flux in the jet, and g is the acceleration of gravity. For $\lambda = \infty$, the jet is slender and parabolic. It becomes thicker as λ decreases, and reaches a a limiting form at $\lambda = 0$. Then there is a stagnation point at the crest,

where the surface makes a 120° angle with itself. This angle is predicted by the same argument Stokes used in his study of water waves.

The problem is formulated as an integro-differential equation for the two free surfaces of the jet. This equation is discretized to yield a set of nonlinear equations which are solved numerically by Newton's method. In addition asymptotic results for λ large are obtained analytically. Graphs of the results are presented.

67. "Fluid dynamics and the Boltzmann equation" by Russel E. Caflisch, Stud. Stat. Mech., to appear.

The fluid dynamic equations and the Boltzmann equation provide alternative descriptions for the evolution of a gas. In the limit of small mean free path, the Euler or Navier-Stokes equations can be derived from the Boltzmann equation through the Hilbert ot Chapman-Enskog expansions. These expansions are singular near boundaries, shocks, and general initial data and special boundary, shock, and initial layers are needed to complete the solution. In the outer region away from these layers, the expansions are shown to be valid, under certain conditions. They approximate a solution of the Boltzmann equation for a reasonable time period, and they provide the dominant term in the long time asymptotics of the solution of the near linear problem. Shock wave solutions of the Boltzmann equation are also discussed. The Boltzmann solution for a weak shock is constructed and is shown to agree with the Navier-Stokes profile to leading order.

68. "Theory of periodic structures in lipid bilayer membranes" by Meira S. Falkovitz, Michael Seul, Harry L. Frisch, and Harden M. McConnell, Proc. Nat. Acad. Sci., submitted.

An approximate, new model for the structure of the periodic, undulated $P_{\beta'}$ phase of phosphatidylcholine bilayers is proposed. The properties of this phase are deduced by minimizing a Landau-de Gennes expression for the bilayer free energy when this free energy contains a term favoring a spontaneous curvature of the membrane. The theoretical calculation leads to a model for the $P_{\beta'}$ phase of phosphatidylcholine bilayers having a number of novel physical properties, including periodic variations in membrane "fluidity."

69. "The fluid-dynamic limit of a model Boltzmann equation in the presence of a shock" by Russel E. Caflisch, Institut National de Recherce en Informatique et en Automatique, INRIA Report No. 81, June 1981, 1-34.

The Broadwell equation is the simplest model of the Boltzmann equation of kinetic theory for which the corresponding model fluid dynamic equations are non-trivial. For this equation there is a complete existence theory for the initial value problem. Here we show formally that if the model fluid dynamic equations can be solved, then the Broadwell solution asymptotically converges to the fluid dynamic solution as the mean free path goes to zero. This limit is valid even if there is a shock in the fluid flow, although there is a thin shock layer in which

the convergence does not hold. Arbitrary smooth initial data is allowed, which leads to a short initial layer of non-convergence, but the initial and shock layers do not interact due to the assumed initial smoothness.

61. "Folds in solutions of two parameter systems and their calculation: Part I" by Allan Jepson and Alastair Spence, Stanford Univ. Numer. Anal. Report, in press.

This paper treats the existence of paths of turning points in solutions of nonlinear systems having two parameters. It is well known that these paths are solutions of a particular extended system of nonlinear equations. In this paper both regular and simple turning points in the extended system are related to the local geometry of the solution surface of the original nonlinear system. A description is given of numerical methods both for solving the extended system and for calculating parameters determining the local geometry of the solution surface. Applications to perturbed bifurcation, to the formation of isolas, and to the calculation of the multiplicity of solutions are also discussed.

IV. WORK IN PROGRESS.

1. "Optimal catalyst distribution in a membrane" (Joseph B. Keller, Meira Falkovitz and Harry Frisch). §It is shown how to distribute a fixed amount of catalyst across the thickness of a plane membrane to maximize the rate of reaction of a substance diffusin through the membrane and reacting within it. The result is that all the catalyst should be located at the front or high concentration face of the membrane. It is also shown how to distribute the catalyst to minimize the flux of unreacted substance out of the membrane. In this case the catalyst should be uniformly distributed in a layer which is symmetric about the midplane of the membrane. The thickness of the layer is determined. The analysis employs the methods of the calculus of variations.

2. "Crawling of worms" (Joseph B. Keller and Meira Falkovitz). §The mechanics of a worm crawling along a flat surface is analyzed. The external forces of friction and gravity, and the internal tension, are taken into account. An equation of motion is formulated, and solutions are sought in which both the tension and the linear density are required to lie between prescribed bounds. Periodic travelling wave solutions on worms of finite length and pulse-like travelling waves on worms of infinite length are treated. The maximum crawling velocity is determined, as well as the wave form which achieves it. The results are compared with experimental observations.

3. "A relation between the Bayes and maximum likelihood methods" (Joseph B. Keller). §The Bayes transformation B_y, based upon an observed value y, converts a prior distribution $p_0(\theta)$ of a parameter θ into a posterior distribution $B_y p_0(\theta)$. J. J. Higgins [Bayesian inference and the optimality of maximum likelihood estimation, Inter. Stat. Rev., 45 (1977), 9-11] has shown that $B_y^n p_0(\theta)$ converges, as n increases, to $[\theta - \theta_1(y)]$ where $\theta_1(y)$ is the unique maximum likelihood estimate of θ based upon y. We show that when there are several maximum likelihood points in the support of $p_0(\theta)$, $B_y^n p_0(\theta)$ converges to a sum of delta functions located at these points. Each is multiplied by a certain coefficient which we determine. This suggests a way of extending the maximum likelihood method to the case of several maximum likelihood points.

4. "Weakly nonlinear geometrical optics" (John K. Hunter and Joseph B. Keller). §A theory of weakly nonlinear solutions of systems of hyperbolic partial differential equations is contructed. The theory leads to propagation of waves along the rays of the corresponding linearized theory. However, the amplitude satisfies a nonlinear equation along these rays. This leads to waveform distortion and shock formation. The theory can be used to describe weak shock waves in any number of dimensions. Its relation to the theory of G. B. Whitham [Linear and Nonlinear Waves, Wiley, 1974] and of L. D. Landau is demonstrated.

5. "Helmholtz resonator modes and scattering" (Harlan B. Sexton and Joseph B. Keller). An integral equation is formulated for the problem of scattering of an acoustic wave by a Helmholtz resonator. The equation is solved asymptotically when the hole is small compared to all other lengths in the problem. Scattering resonances are found to occur at certain complex frequencies of the incident wave. These are shown to be the frequencies of the modes of the resonator. These modes are determined asymptotically. The corresponding results are also developed for the two-dimensional problem which represents water waves incident upon the narrow mouth of a harbor.

6. "Water wave production by an oscillating body" (Joseph B. Keller and Si-Xiong Chen). §Asymptotic methods are employed to calculate the waves generated by an oscillating body floating on the surface of a fluid. The method of geometrical optics is used to describe the far field, and boundary layer methods are used to describe the near field. The results generalize those of other authors.

7. "Reflection, transmission, absorption and scattering of acoustic waves by rough surfaces" (John G. Watson and Joseph B. Keller). §The first and second moments, i.e. the coherent field and the two-point two-time correlation function, are calculated for the acoustic fields scattered from various rough surfaces. For each surface they yield the reflection, transmission, absorption and differential scattering coefficients, as well as an equivalent boundary condition for the coherent

field. Renormalized coefficients are constructed to eliminate divergences at grazing incidence. The results are specialized to surfaces which are statistically homogeneous in both space and time, to surfaces which are not moving, to surfaces which are simply or multiply periodic, and to surfaces consisting of randomly placed bosses on a smooth surface. The surfaces considered are slightly rough, moving, soft or hard boundaries, and flat surfaces with random admittance or impedance. The analysis is based on the regular perturbation method. Comparisons with previous results are made.

8. "The product-expansion technique" (John G. Watson). §We present a method, the product-expansion technique, which is an alternative to the smoothing method. Given equations for a random process, the technique produces an approximate solution for the expected value in the form of the product of averages of operator products. An application to rough surface acoustic scattering is given.

9. "Branches of periodic solutions and their computation" (Allan D. Jepson, Esabius Doedel and Herbert B. Keller). §Branches of periodic solutions to autonomous systems of differential equations with a free parameter are studied. A bifurcation analysis is presented with a view towards numerical methods. A computational package for following these branches, detecting bifurcation points, switching branches at simple bifurcation points, and switching branches at Hopf bifurcation points is tested on several examples.

10. "Asymptotic boundary conditions for ordinary differential equations, Parts I and II" (Allan D. Jepson and Herbert B. Keller). §The numerical solution of two point boundary value problems on semi-infinite intervals is often obtained by truncating the interval at some finite point. In this paper, we determine a hierarchy of increasingly accurate boundary conditions for the truncated interval problem. Both linear and nonlinear problems are considered. Numerical techniques for error estimation and the determination of an appropriate truncation point are discussed. Several numerical examples are given.

11. "Fredholm theories for two point boundary value problems on semi-infinite intervals" (Allan D. Jepson). §A simple method to determine the Fredholm properties of two point boundary value problems on semi-infinite intervals is presented. The general problem is reduced to an equivalent regular problem on a finite interval. The Fredholm properties then follow from the well known theory for regular problems and from asymptotic estimates on particular solutions of the singular problem.

12. "Paths of simple bifurcation points and their computation" (Allan D. Jepson and Alastair Spence). §Numerical methods for following simple bifurcation points in nonlinear systems with three parameters are presented. Critical points in these branches are studied.

14. "Bifurcation and stability in constrained variational problems" (John H. Maddocks). §The main result presented is a theorem determining when a quadratic form subject to linear constraints is positive-definite. Such quadratic forms arise as tests for stability in general constrained variational problems. Connections with bifurcation and exchange of stability are also described.

15. "Stability of the elastica" (John H. Maddocks). §The elastica is a standard model for the buckling of an elastic rod. This paper determines stability properties of various planar equilibria of a nonlinearly elastic rod, both to variations in the plane and out of the plane. Previously undiscovered second bifurcations with associated loss of stability are described.